by : rraezzz

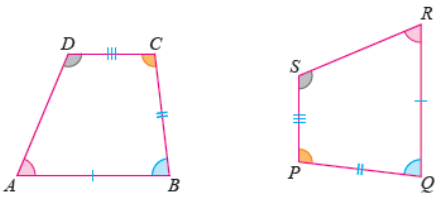
CHAPTER 5

congruency, enlargement and combined tranformations

by : raezzz

5.1 CONGRUENCY

❑ two congruent polygons have the same measurement for the corresponding sides and angles

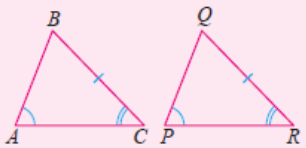


In the diagram above, quadrilaterals $ABCD$ and $PQRS$ are congruent with the conditions shown in the following table.

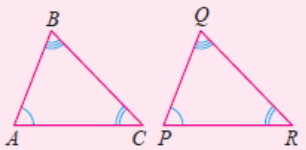
Lengths of corresponding sides	Sizes of corresponding angles
$AB = RQ$ $BC = QP$ $CD = PS$ $AD = RS$	$\angle BAD = \angle QRS$ $\angle ABC = \angle RQP$ $\angle BCD = \angle QPS$ $\angle ADC = \angle RSP$

❑ triangle congruency

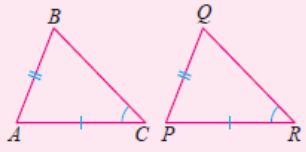
Angle-Angle-Side (AAS)
- Two corresponding angles are equal and the length of one of the corresponding sides which does not lie between the two angles are equal
- $\angle BAC = \angle QPR$
  $\angle ACB = \angle PRQ$
  $BC = QR$



Angle-Angle-Angle (AAA)
- All the three corresponding angles are equal
- The area of the pair of triangles must be equal
- $\angle BAC = \angle QPR$
  $\angle ACB = \angle PRQ$
  $\angle ABC = \angle PQR$



Side-Side-Angle (SSA)
- Two corresponding sides are equal in length and one of the corresponding angles which is not subtended between the two sides are equal
- The area of the pair of triangles must be equal
- $AC = PR$, $AB = PQ$, $\angle ACB = \angle PRQ$



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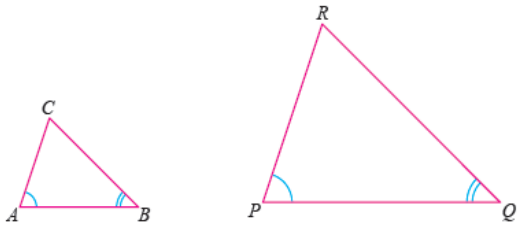
5.2 ENLARGEMENT

❑ similarity of geometric objects

The results of Mind Mobilisation 3 show that two geometric objects are **similar** when

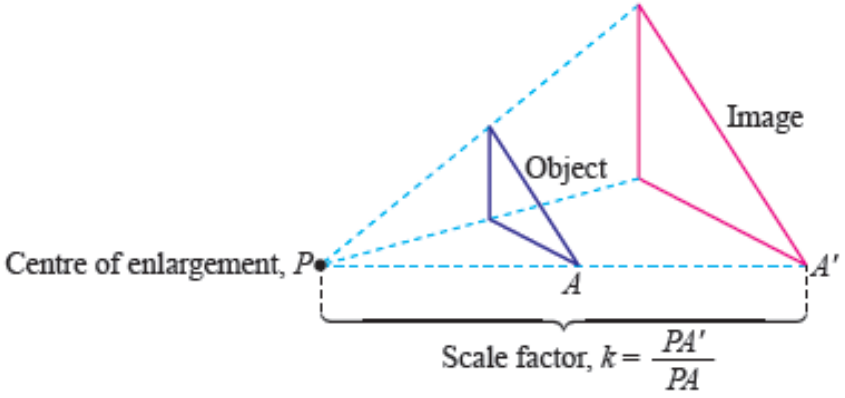
- all the corresponding angles are equal, namely $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$
- all the ratios of corresponding sides are equal, namely $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$



Smart TIPS

If all the corresponding sides of a pair of triangles are proportional, then all the corresponding angles are equal and vice versa.

- ❑ enlargement is a transformation in which all the points of an object move from fixed point with a constant ratio
- ❑ the fixed point is knows as the centre of enlargement and the constant ratio is known as the scale factor



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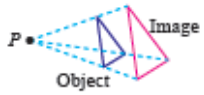
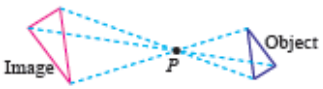
5.2 ENLARGEMENT

❑ Scale factor

$$k = \frac{\text{distance of point of image from } P}{\text{distance of point of object from } P} \quad \text{or} \quad k = \frac{\text{length of side of image}}{\text{length of side of object}}$$

Scale factor, k	Size of image	Position of image to the centre of enlargement P	
$k > 1$	**Larger** than the object	Be on the same side as the object	
$k = 1$	**Equal** in size to the object	Be on the same side as the object	
$0 < k < 1$	**Smaller** than the object	Be on the same side as the object	
$-1 < k < 0$	**Smaller** than the object	Be on the opposite side of the object	
$k = -1$	**Equal** in size to the object	Be on the opposite side of the object	
$k < -1$	**Larger** than the object	Be on the opposite side of the object	

❑ Relation between area of the image and area of object of an enlargement

The results of Mind Mobilisation 5 show that $\frac{\text{area of the image}}{\text{area of the object}} = k^2$ where k is the scale factor.

Hence, we can determine the area of the image of an enlargement with the formula:

$$\text{Area of the image} = k^2 \times \text{Area of the object}$$

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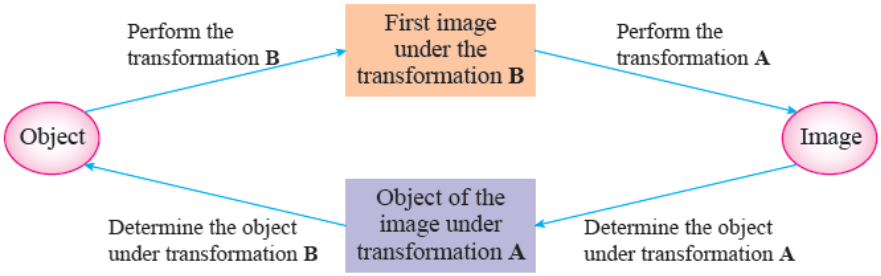
5.3 COMBINED TRANSFORMATION

❑ Combined tranformations AB means transformation B followed by transformation A

The diagram below shows the steps to determine the image or object of a combined transformation **AB**.

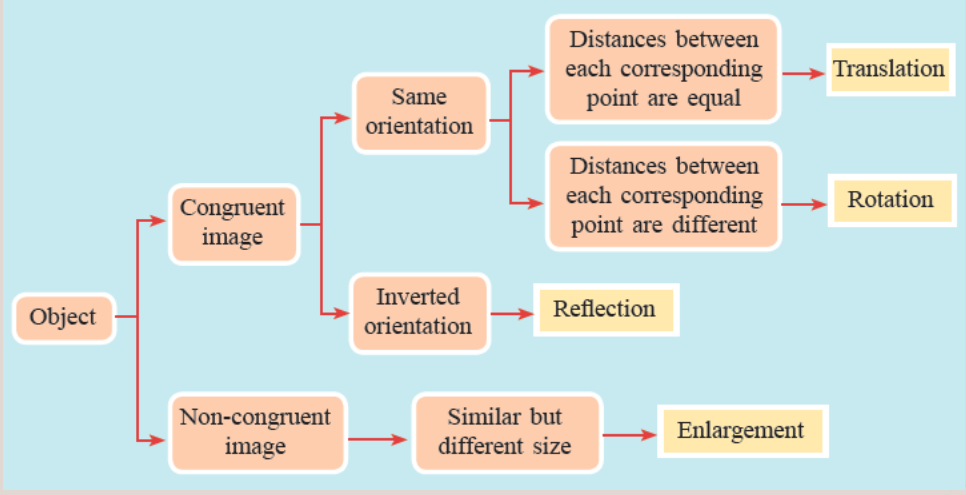

Perform the transformation **B**
First image under the transformation **B**
Perform the transformation **A**
Object
Image
Object of the image under transformation **A**
Determine the object under transformation **B**
Determine the object under transformation **A**

❑ Effect of commutative law on combined transformation

In general,

- A combined transformation **AB** satisfies the commutative law if the images under the combined transformation **AB** and combined transformation **BA** are the same.
- A combined transformation **AB** does not satisfy the commutative law if the images under the combined transformation **AB** and combined transformation **BA** are not the same.

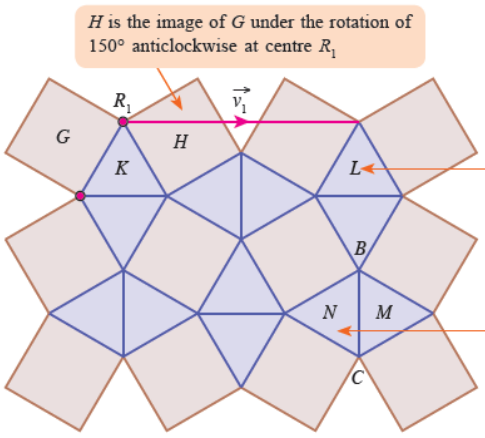
❑ describe the combined transformation



5.4 TESSELLATION

- Tessellation is a pattern of recurring shapes that fills a plane without leaving empty spaces or overlapping

The following shows some steps involved in producing the tessellation in the activity above.



MEMORY BOX

Isometric transformation is the transformation that produces an image that is congruent with an object.

Example 22

The diagram below shows a tessellation consisting of equilateral triangles which are produced by isometric transformation. State the transformation involved to produce

(a) shape Q from shape P
(b) shape R from shape Q
(c) shape S from shape P

Solution: